Uncertainty Quantification in Performance Evaluation of Manufacturing Processes

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Why Uncertainty Quantification?

- Performance evaluation – affected by available data and uncertainty
- Uncertainty sources – linear or non-linear combination, occur at different stages
- Information sources – heterogeneous
- A systematic approach is necessary for information fusion and uncertainty quantification.
Aleatory vs. epistemic uncertainty

Four types of quantities

1. Deterministic $\rightarrow$ **known** value (fixed constant)

2. Stochastic $\rightarrow$ random variability
   - Distribution statistics are **known**
   - **Aleatory**, irreducible uncertainty

3. Quantity is deterministic, but **unknown**
   - **Epistemic**, reducible uncertainty

4. Quantity is stochastic, but distribution characteristics are **unknown**
   - Has both **aleatory** and **epistemic** uncertainty
     - Unknown distribution type
     - Unknown distribution parameters
Bayesian network

Joint PDF of all variables
\[ \pi(U) = \pi(a) \times \pi(b|a) \times \pi(c|a) \times \pi(d|c) \times \pi(e|b,d) \times \pi(f) \times \pi(g|e,f) \]

PDF of node \( g \)
\[ \pi(g) = \int \pi(U) \, da \, db \ldots df \]

With new observed data \( m \)
\[ \pi(U, m) = \pi(U) \times \pi(m|b) \]
Methodology for UQ (1/2)

Construction of Bayesian Network

- Physics-based Models
  - Use available models based on process physics (domain knowledge)
    Eg: Differential equations

- Data-driven approach
  - Build surrogate models using data
    e.g., Regression, Polynomial models, Gaussian Process
  - Structure Learning algorithms
    Constraint-based and Score-based algorithms

- Hybrid approach
  - Combination of physics-based and data-driven approaches
  - Partial structure of the BN using domain knowledge
  - Learn other parts of network using data

Bartram & Mahadevan, SCHM, 2014
Methodology for UQ (2/2)

Bayesian Model Calibration/Updating

- Estimate unknown (epistemic) parameters using data in a probabilistic manner

- Bayes’ theorem

\[
Pr(\bar{N}_{obs} | N_{obs} = D) = \frac{Pr(N_{obs} = D | \bar{N}_{obs}) Pr(\bar{N}_{obs})}{\int Pr(\bar{N}_{obs}, N_{obs} = D) d\bar{N}_{obs}}
\]

Sampling from Posterior: Markov Chain Monte Carlo (MCMC)

Uncertainty Propagation

- Obtain updated posterior distribution of the Quantity of Interest (QoI)

- Monte Carlo Sampling - samples from posterior distribution

- Samples are propagated through the network

Posterior distribution of QoI (Performance Metric)
Scalability → Dimension Reduction

Size of Production Network increases

- Increase in Epistemic Parameters
- Increase in Computational effort

Goal: Obtain a reduced set of variables such that there is no significant change in statistics of QoI

Approach: Global Sensitivity Analysis

Assess the variance contribution of each of the variables to the variance of the QoI.

\[
Y = G(X_1, X_2 \ldots X_n) \\
S_i^L = \frac{Var_{X_i}(E_{X-i}(Y|X_i))}{Var(Y)} \\
S_i^T = 1 - \frac{E_{X-i}(Var_{X_i}(Y|X_i))}{Var(Y)}
\]

- Obtain prior sensitivity indices by performing sensitivity analysis using prior distributions
- Assume a threshold value for sensitivity index.
- If sensitivity index < threshold value, assume that variable to be deterministic at the nominal value.
Sensitivity Analysis under Epistemic Uncertainty

- Global Sensitivity Analysis
  - Applicable to $Y = G(X)$ where $G$ is deterministic
  - One value of $X \Rightarrow$ One value of $Y$
  - Define auxiliary variables
  - Can include both aleatory & epistemic sources

- Data uncertainty

- Model uncertainty

- Introduce auxiliary variable $U \Rightarrow$ represent variability $\Rightarrow U (0, 1)$

\[ U = \int_{-\infty}^{X} f_X(x \mid P) \, dx \]

Sankararaman & Mahadevan, RESS, 2013
Handling Big Data

- HDFS: Hadoop Distributed File System
  - To store large amounts of data in databases

- MapReduce
  - A parallel processing framework for large-scale data processing.

- Model Building using all data → computationally unaffordable

- Data Reduction and Feature Selection techniques required
  - Clustering
  - Selection of Data using Design of Experiments (DoE)
  - Apache Mahout: Machine learning library for Hadoop

- The reduced dataset can be used for Bayesian Network construction
Summary – Methodology

Without Dimension/Data Reduction

Construction of Bayesian Network

Bayesian Model Calibration, Updating

Uncertainty Propagation

With Data/Dimension Reduction

Data Reduction, Dimension Reduction

Construction of Bayesian Network

Bayesian Model Calibration, Updating

Uncertainty Propagation

With Data/Dimension Reduction
Demonstration Problem

Injection Molding

Die Casting

Process

Machine1

Machine2

Machine3

Turning

Metal Part

Arc Welding

GTAW

OR

GMAW

Metal Part

Metal Module

Threaded Fastening

e.g. Power Train

Product Module

Metallic Module

Plastic Part

Plastic Grains

Metal Powder

Product

Machine3

Machine2

Machine1

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UQ in Performance Evaluation of Manufacturing Processes
Presenter: Sankaran Mahadevan
Injection Molding

Goal:

UQ in Energy Consumption of an Injection Molding Process

Three stages

- Melting of polymer
- Injection of polymer into the mold
- Cooling of polymer to form product

Melting Process

- Power used for melting the dye
  \[ P_{melt} = \rho \times Q_{avg} \times C_P \times (T_{inj} - T_{pol}) + \rho \times Q_{avg} \times H_f \]

- Volume of shot
  \[ V_{shot} = V_{part} \times (1 + \frac{\epsilon}{100} + \frac{\Delta}{100}) \]

- Energy consumed for melting
  \[ E_{melt} = \frac{P_{melt} \times V_{shot}}{Q} \]

- Flow rate and Average flow rate
  \[ Q_{avg} = 0.5 \times Q \]

Legend

- \( \rho \) = specific gravity of polymer
- \( C_P \) = heat capacity of polymer
- \( T_{inj} \) = Injection temperature
- \( T_{pol} \) = polymer temperature
- \( H_f \) = polymer heat of fusion
- \( V_{part} \) = Volume of mold
- \( \epsilon \) = shrinkage
- \( \Delta \) = buffer
- \( V_{shot} \) = total volume injected
Injection Process

- Injection time
  \[ t_{inj} = \frac{2 \times V_{part} \times [1 + \epsilon + \Delta] \times n \times p_{inj}}{1000 \times P_{inj}} \]

- Energy consumed in injection process
  \[ E_{inj} = p_{inj} \times V_{part} \]

Legend

\( n = \text{number of cavities} \)
Cooling Process

• Cooling Time

\[ t_{cool} = \left( \frac{h_{\text{max}}^2}{\pi^2 \gamma} \right) \times \ln \left( \frac{4}{\pi} \times \frac{T_{\text{inj}} - T_m}{T_{\text{ej}} - T_m} \right) \]

• Energy consumed in cooling process

\[ E_{\text{cool}} = \frac{\rho \times V_{\text{part}} \times \left[ C_p \times (T_{\text{inj}} - T_{\text{ej}}) \right]}{\text{COP}} \]

Legend

\( h_{\text{max}} \) = maximum wall thickness of mold
\( \gamma \) = thermal diffusivity
\( T_{\text{inj}} \) = Injection temperature
\( T_{\text{inj}} \) = Injection temperature
\( T_{\text{inj}} \) = Injection temperature
\( T_{\text{inj}} \) = Injection temperature
\( T_{\text{ej}} \) = ejection temperature
\( \text{COP} \) = coefficient of performance of cooling equipment
Total Energy Consumption

• Energy consumed in making a part

\[ E_{part} = \frac{1}{n} \times \left( \frac{0.75 \times E_{melt} + E_{inj}}{\eta_{inj}} + \frac{E_{reset}}{\eta_{reset}} + \frac{E_{cool}}{\eta_{cool}} + \frac{0.25 \times E_{melt}}{\eta_{heater}} \right) \times \frac{n \times (1 + \epsilon + \Delta)}{\eta_{machine}} + P_b \times T_{total} \]

• Total Electricity Cost

\[ C_{energy} = \frac{(E_{part} \times N_{day} \times D_{day} \times U_{C_{USA}})}{3600 \times 1000} \]

• Reset Energy

\[ E_{reset} = 0.25(E_{inj} + E_{cool} + E_{melt}) \]

• Total Cycle Time

\[ t_{cycle} = t_{inj} + t_{cool} + t_{reset} \]

• Reset time

\[ t_{reset} = 1 + 1.75t_d \sqrt{\frac{2d+5}{s}} \]

Legend

\( h_{\text{max}} = \) maximum wall thickness of mold
\( E_{reset} = \) Energy consumed for resetting
\( P_b = \) Power required for basic energy consuming units, when machine is in stand-by mode
\( t_d = \) dry cycle time
\( s = \) maximum clamp stroke in the mold
\( U_{C_{USA}} = \) unit cost in USA

\( T_{total} = \) Total cycle time
\( \eta_{inj}, \eta_{reset}, \eta_{cool}, \eta_{heater}, \eta_{machine} = \) Efficiencies of injection, reset, cooling, heating, machine power
\( d = \) depth of the part
\( N_{day} = \) output per day
\( D_{day} = \) number of days
Injection Molding – Bayesian Network
Injection Molding Materials

Several types of materials used in injection molding

- Metals
- Glass
- Elastomers
- Thermoplastic polymers
- Thermosetting polymers

Polyethylene properties (assumed)

- Specific gravity $\rho = 960 \text{ kg/m}^3$
- Heat capacity $C_p = 2250 \text{ J/kg-K}$
- Thermal diffusivity $= 2.27 \times 10^{-7} \text{ m}^2/\text{s}$
- Shrinkage $\epsilon = 0.019$
- Heat of fusion $H_f = 240 \text{ kJ/kg}$
## Injection Molding Process Parameters – Synthetic Dataset

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection temperature $T_{inj}$</td>
<td>210 (°C)</td>
</tr>
<tr>
<td>Mold temperature $T_m$</td>
<td>35 (°C)</td>
</tr>
<tr>
<td>Ejection temperature $T_{ej}$</td>
<td>50 (°C)</td>
</tr>
<tr>
<td>Injection pressure $p_{inj}$</td>
<td>90 MPa</td>
</tr>
<tr>
<td>Flow rate $Q$</td>
<td>1.67e-5 $m^3/s$</td>
</tr>
<tr>
<td>Coefficient of performance $COP$</td>
<td>0.7</td>
</tr>
<tr>
<td>All efficiency coefficients $\eta$</td>
<td>0.7</td>
</tr>
<tr>
<td>Number of cavities $n$</td>
<td>1</td>
</tr>
<tr>
<td>Fraction $\Delta$</td>
<td>0.015</td>
</tr>
<tr>
<td>Thickness $h_{max}$</td>
<td>0.0125 m</td>
</tr>
<tr>
<td>Volume of part $V_{part}$</td>
<td>0.002048 $m^3$</td>
</tr>
<tr>
<td>Unit Cost $UC_{USA}$</td>
<td>10 cents/kWh</td>
</tr>
<tr>
<td>Parts per day $N_{day}$</td>
<td>10,000</td>
</tr>
<tr>
<td>Number of days $D_{day}$</td>
<td>28</td>
</tr>
</tbody>
</table>
Injection Molding Process - Synthetic dataset

- Error in temperature measurements (°C)
  \[ T_{obs} = T_{true} + \epsilon_T \sim N(0,3) \]
- Error in cycle time measurements (s)
  \[ t_{obs} = t_{true} + \epsilon_t \sim N(0,2) \]
- Error in injection pressure measurements (MPa)
  \[ p_{inj_{obs}} = p_{inj_{true}} + \epsilon_{p_{inj}} \sim N(0,4) \]
- Initial temperature of polymer (°C)
  \[ T_{pol} = N(30,2) \]
- Density of polymer (kg/m³)
  \[ \rho = N(960,10) \]
- Heat Capacity (J/kg-K)
  \[ H_f = N(2250,20) \]
- Flow Rate (m³/s)
  \[ Q = Q_{true} + \epsilon_Q \sim N(0,1.5e-6) \]

100 data points are generated and used in calibration process

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{inj} ) (°C)</td>
<td>Uniform (205,220)</td>
</tr>
<tr>
<td>( T_{ej} ) (°C)</td>
<td>Uniform (45,60)</td>
</tr>
<tr>
<td>( T_m ) (°C)</td>
<td>Uniform (30,45)</td>
</tr>
<tr>
<td>( p_{inj} ) (MPa)</td>
<td>Uniform (88,95)</td>
</tr>
<tr>
<td>( Q ) (m³/s)</td>
<td>Uniform (1.6e-5,1.75e-5)</td>
</tr>
</tbody>
</table>
### Results – Prior Sensitivity Analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type of Uncertainty</th>
<th>Prior – Individual effect</th>
<th>Prior - Total effect</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{inj}$</td>
<td>Epistemic</td>
<td>0.4857</td>
<td>0.5829</td>
<td>Significant</td>
</tr>
<tr>
<td>$T_{pol}$</td>
<td>Aleatory</td>
<td>0.0161</td>
<td>0.0363</td>
<td>Significant</td>
</tr>
<tr>
<td>$T_{ej}$</td>
<td>Epistemic</td>
<td>0.2412</td>
<td>0.3617</td>
<td>Significant</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Aleatory</td>
<td>0.1226</td>
<td>0.1264</td>
<td>Significant</td>
</tr>
<tr>
<td>$P_{inj}$</td>
<td>Epistemic</td>
<td>0.0028</td>
<td>0.0078</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Aleatory</td>
<td>0.0015</td>
<td>0.0035</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Aleatory</td>
<td>0.0134</td>
<td>0.0137</td>
<td>Significant</td>
</tr>
</tbody>
</table>

Sensitivity index < 0.01 (Threshold value) → Insignificant

Variables retained for Calibration: Injection Temperature, Ejection Temperature
Results - Prior and Posterior Plots (Epistemic variables)

**Injection Temperature**

**Ejection Temperature**

**Energy consumed per part**
## Results – Prior and Posterior Sensitivity Analysis

<table>
<thead>
<tr>
<th>Variable</th>
<th>Type of Uncertainty</th>
<th>Prior – Individual effect</th>
<th>Prior – Total effect</th>
<th>Remarks</th>
<th>Posterior – Individual effect</th>
<th>Posterior – Total effect</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{inj}$</td>
<td>Epistemic</td>
<td>0.4857</td>
<td>0.5829</td>
<td>Significant</td>
<td>0.0083</td>
<td>0.0125</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$T_{pot}$</td>
<td>Aleatory</td>
<td>0.0161</td>
<td>0.0363</td>
<td>Significant</td>
<td>0.0686</td>
<td>0.0961</td>
<td>Significant</td>
</tr>
<tr>
<td>$T_{ej}$</td>
<td>Epistemic</td>
<td>0.2412</td>
<td>0.3617</td>
<td>Significant</td>
<td>0.0063</td>
<td>0.0126</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Aleatory</td>
<td>0.1226</td>
<td>0.1264</td>
<td>Significant</td>
<td>0.7997</td>
<td>0.818</td>
<td>Significant</td>
</tr>
<tr>
<td>$P_{inj}$</td>
<td>Epistemic</td>
<td>0.0028</td>
<td>0.0078</td>
<td>Insignificant</td>
<td>0.0018</td>
<td>0.0019</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Aleatory</td>
<td>0.0015</td>
<td>0.0035</td>
<td>Insignificant</td>
<td>0.0107</td>
<td>0.0418</td>
<td>Significant</td>
</tr>
<tr>
<td>$C_{p}$</td>
<td>Aleatory</td>
<td>0.0134</td>
<td>0.0137</td>
<td>Significant</td>
<td>0.0414</td>
<td>0.0798</td>
<td>Significant</td>
</tr>
</tbody>
</table>

Uncertainty in $T_{inj}$ and $T_{ej}$ greatly reduced after calibration process
(Effects $\rightarrow$ Significant to Insignificant)
Goal: UQ in Energy Consumption of a Welding Process

• Volume of the Weld

\[ V = L \times \left( \frac{3}{4} \times lh + gt + \frac{(l - g)}{2} \times (t - e) \right) \]

• Theoretical Minimum Energy (filler and metal are the same)

\[ E_{AP}^{TR} = \rho \left( C_p (T_f - T_i) + H \right) V \]

• Input Energy (Laser)

\[ E_{AP}^{I} = U I \left( \frac{L}{S} \right) \]

• Efficiency of welding process

\[ \eta_{AP} = \frac{E_{AP}^{TR}}{E_{AP}^{I}} \]

Legend

- \( L \): Length of weld
- \( \rho \): density of material
- \( C_p \): Heat capacity of material
- \( T_f \): Final Temperature
- \( T_i \): Initial Temperature
- \( H \): Latent Heat
- \( U \): Voltage
- \( I \): Current
- \( S \): Welding Speed
Energy Consumption in Welding Process (2/2)

- Total Power

\[
P_{total} = \frac{E_{robot}}{t_{robot}} + \frac{E_{AP}}{t_{ps}} + \frac{E_{cooling}}{t_{cooling}}
\]

- Total Electricity Cost

\[
EC = P_{total} \times t_{total} \times D_{day} \times UC_{USA}
\]

Legend

- \( t_{total} \) = Total weld time
- \( D_{day} \) = Number of days in service
- \( UC_{USA} \) = Unit cost for electricity in USA
Welding Process – Bayesian Network

- **Constants**
- **Calibration Parameters (Epistemic)**
- **Deterministic**
- **Aleatory**
- **Observations**
Synthetic Dataset (1/2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Temperature ($T_i$) (K)</td>
<td>$N(303,0.3)$</td>
</tr>
<tr>
<td>Final Temperature ($T_f$) (K)</td>
<td>$N(1628,10)$</td>
</tr>
<tr>
<td>Heat Capacity ($C_p$) (J/kgK)</td>
<td>$N(500,5)$</td>
</tr>
<tr>
<td>Density ($\rho$) (kg/m$^3$)</td>
<td>$N(8238,10)$</td>
</tr>
<tr>
<td>Latent Heat ($H$) (kJ/kg)</td>
<td>$N(270,3)$</td>
</tr>
<tr>
<td>Weld Zone parameters (mm)</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>$N(8.5,0.5)$</td>
</tr>
<tr>
<td>$h$</td>
<td>$N(2.6,0.5)$</td>
</tr>
<tr>
<td>$g$</td>
<td>$N(2,0.1)$</td>
</tr>
<tr>
<td>$t$</td>
<td>$N(15,0.5)$</td>
</tr>
<tr>
<td>$e$</td>
<td>$N(11,1)$</td>
</tr>
<tr>
<td>Length of weld ($L$) (mm)</td>
<td>$N(500,10)$</td>
</tr>
<tr>
<td>Voltage ($U$) (V)</td>
<td>20</td>
</tr>
<tr>
<td>Current ($I$) (A)</td>
<td>250</td>
</tr>
<tr>
<td>Weld Speed ($S$) (mm/min)</td>
<td>700</td>
</tr>
<tr>
<td>$D_{day}$</td>
<td>26</td>
</tr>
<tr>
<td>$UC_{USA}$</td>
<td>10 cents/kWh</td>
</tr>
</tbody>
</table>
Synthetic Dataset (2/2)

- Error in length measurement (mm), $\epsilon_m \sim N(0,0.01)$
- Error in current measurement (A), $\epsilon_c \sim N(0,2)$

**Observation Data**

- $l_{obs} = l + \epsilon_m = N(8.5,0.5) + N(0,0.01)$
- $h_{obs} = h + \epsilon_m = N(2.6,0.5) + N(0,0.01)$
- $e_{obs} = e + \epsilon_m = N(11,1) + N(0,0.01)$
- $I_{obs} = I + \epsilon_c = 250 + N(0,2)$

**Prior Distributions**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>$\mu_l$ $\sigma_l$</td>
</tr>
<tr>
<td>$h$</td>
<td>$\mu_h$ $\sigma_h$</td>
</tr>
<tr>
<td>$e$</td>
<td>$\mu_e$ $\sigma_e$</td>
</tr>
<tr>
<td>$I$</td>
<td>$\sigma_c$</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td></td>
</tr>
</tbody>
</table>

$\sigma_c$ - Standard deviation in measurement error of current
## Results – Prior Sensitivity Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type of uncertainty</th>
<th>Prior – Individual effect</th>
<th>Prior – Total effect</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_l$</td>
<td>Aleatory</td>
<td>0.0745</td>
<td>0.0872</td>
<td>Significant</td>
</tr>
<tr>
<td>$\mu_l$</td>
<td>Epistemic</td>
<td>0.0026</td>
<td>0.0027</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Epistemic</td>
<td>$6.44 \times 10^{-7}$</td>
<td>$7.75 \times 10^{-3}$</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$u_h$</td>
<td>Aleatory</td>
<td>0.2147</td>
<td>0.2238</td>
<td>Significant</td>
</tr>
<tr>
<td>$\mu_h$</td>
<td>Epistemic</td>
<td>0.0073</td>
<td>0.0074</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>Epistemic</td>
<td>$7.056 \times 10^{-8}$</td>
<td>0.00826</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$u_e$</td>
<td>Aleatory</td>
<td>0.3153</td>
<td>0.3227</td>
<td>Significant</td>
</tr>
<tr>
<td>$\mu_e$</td>
<td>Epistemic</td>
<td>0.2019</td>
<td>0.2032</td>
<td>Significant</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>Epistemic</td>
<td>$1.225 \times 10^{-6}$</td>
<td>0.00569</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$g$</td>
<td>Aleatory</td>
<td>0.044</td>
<td>0.0442</td>
<td>Significant</td>
</tr>
<tr>
<td>$t$</td>
<td>Aleatory</td>
<td>0.1651</td>
<td>0.1655</td>
<td>Significant</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Aleatory</td>
<td>0.000124</td>
<td>0.000125</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Aleatory</td>
<td>0.00435</td>
<td>0.0044</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$H$</td>
<td>Aleatory</td>
<td>0.00091</td>
<td>0.00093</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Aleatory</td>
<td>$2.393 \times 10^{-6}$</td>
<td>$2.421 \times 10^{-6}$</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Aleatory</td>
<td>0.00225</td>
<td>0.00228</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$L$</td>
<td>Aleatory</td>
<td>0.0347</td>
<td>0.0351</td>
<td>Significant</td>
</tr>
</tbody>
</table>

Threshold value – 0.01

Significant Epistemic variables - $\mu_e$
Results – Prior and Posterior Plots (Epistemic Variables)

Weld parameter ‘e’

Prior and Posterior distributions of $\mu_x$

Prior and Posterior distributions of Theoretical Energy

Theoretical energy consumption per weld

Prior and Posterior $\rightarrow$ no major change, most of the uncertain variables are aleatory (uncertainty can not be reduced)
### Results – Prior and Posterior Sensitivity Analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type of uncertainty</th>
<th>Prior – Individual effect</th>
<th>Prior – Total effect</th>
<th>Remarks</th>
<th>Posterior – Individual effect</th>
<th>Posterior – Total effect</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l$</td>
<td>$u_l$ Aleatory</td>
<td>0.0745</td>
<td>0.0872</td>
<td>Significant</td>
<td>0.1193</td>
<td>0.1226</td>
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</tr>
<tr>
<td></td>
<td>$\mu_l$ Epistemic</td>
<td>0.0026</td>
<td>0.0027</td>
<td>Insignificant</td>
<td>0.00012</td>
<td>0.00013</td>
<td>Insignificant</td>
</tr>
<tr>
<td></td>
<td>$\sigma_l$ Epistemic</td>
<td>$6.44 \times 10^{-7}$</td>
<td>$7.75 \times 10^{-3}$</td>
<td>Insignificant</td>
<td>$1.062 \times 10^{-7}$</td>
<td>$6.005 \times 10^{-5}$</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$h$</td>
<td>$u_h$ Aleatory</td>
<td>0.2147</td>
<td>0.2238</td>
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<td>0.2995</td>
<td>0.3001</td>
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</tr>
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<td>$\mu_h$ Epistemic</td>
<td>0.0073</td>
<td>0.0074</td>
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<td>0.00025</td>
<td>0.000247</td>
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<td>$\sigma_h$ Epistemic</td>
<td>$7.056 \times 10^{-8}$</td>
<td>$0.00826$</td>
<td>Insignificant</td>
<td>$7.208 \times 10^{-10}$</td>
<td>$0.000135$</td>
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<tr>
<td>$e$</td>
<td>$u_e$ Aleatory</td>
<td>0.3153</td>
<td>0.3227</td>
<td>Significant</td>
<td>0.2967</td>
<td>0.2989</td>
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<tr>
<td></td>
<td>$\mu_e$ Epistemic</td>
<td>0.2019</td>
<td>0.2032</td>
<td>Significant</td>
<td>0.00034</td>
<td>0.00034</td>
<td>Insignificant</td>
</tr>
<tr>
<td></td>
<td>$\sigma_e$ Epistemic</td>
<td>$1.225 \times 10^{-6}$</td>
<td>$0.00569$</td>
<td>Insignificant</td>
<td>$2.991 \times 10^{-9}$</td>
<td>$0.000147$</td>
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<tr>
<td>$g$</td>
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<td>0.0493</td>
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<td>$t$</td>
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<td>0.1655</td>
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<td>0.2069</td>
<td>0.2075</td>
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<td>$\rho$</td>
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<td>0.000153</td>
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<tr>
<td>$C_p$</td>
<td>Aleatory</td>
<td>0.00435</td>
<td>0.0044</td>
<td>Insignificant</td>
<td>0.00549</td>
<td>0.00554</td>
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</tr>
<tr>
<td>$H$</td>
<td>Aleatory</td>
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<td>0.00093</td>
<td>Insignificant</td>
<td>0.00107</td>
<td>0.00108</td>
<td>Insignificant</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Aleatory</td>
<td>$2.393 \times 10^{-6}$</td>
<td>$2.421 \times 10^{-6}$</td>
<td>Insignificant</td>
<td>$2.054 \times 10^{-6}$</td>
<td>$2.564 \times 10^{-6}$</td>
<td>Insignificant</td>
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<tr>
<td>$T_f$</td>
<td>Aleatory</td>
<td>0.00225</td>
<td>0.00228</td>
<td>Insignificant</td>
<td>0.00313</td>
<td>0.00316</td>
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</tr>
<tr>
<td>$L$</td>
<td>Aleatory</td>
<td>0.0347</td>
<td>0.0351</td>
<td>Significant</td>
<td>0.04097</td>
<td>0.0413</td>
<td>Significant</td>
</tr>
</tbody>
</table>

Uncertainty in $\mu_e$ greatly reduced after calibration process (Significant to Insignificant)
Comprehensive framework for uncertainty integration and management

Bayesian network
- Include all available models and data
- Include calibration, verification and validation results at multiple levels
- Heterogeneous data of varying precision and cost
- Models of varying complexity, accuracy, cost
- Data on different but related systems

Facilitates
- Forward problem: UQ in overall system-level prediction
  - Integrate all available sources of information and results of modeling/testing/monitoring
- Inverse problem: Resource allocation at various stages of system life cycle
  - Model development, data collection, system design, manufacturing, operations, health monitoring, risk management
Summary of Analyses

• Information Retrieval– retrieve required information from database
• Dimension Reduction – Select a subset of features/parameters
• Data Reduction – Reduce amount of data through clustering
• Model Building – Build a Bayesian Network
• Uncertainty Propagation – Monte Carlo simulation
• Sensitivity Analysis (Aleatory vs. Epistemic) – Variance decomposition
• Performance Prediction – Monte Carlo simulation
• Temporal variation -- Dynamic Bayesian Network
• Diagnosis, Prognosis – Classification, Particle filtering
• Decision-Making, Resource allocation
Future Work

- Derive Bayesian network from process and model libraries
- Include text and image data, along with numerical data in UQ
- Apply to production network with multiple processes
- Analysis over time with streaming data (Dynamic Bayesian Network)
- Explore various problems in manufacturing
  - Process Monitoring (Dynamic Tracking) – For diagnosis
  - Stochastic Process Optimization – Adjust parameters to meet the requirements
  - Risk Management – Reduce performance variability to be within desired limits
  - Resource Allocation – maximize information \( \rightarrow \) maximize reduction in uncertainty